Physics 232 – Boltzmann's Constant

last modified March 31, 2023

1 Introduction

The purpose of this experiment is to measure the ratio of two fundamental physical constants e/k_B using a simple transistor circuit. The quantity e is the magnitude of the electron charge and k_B is Boltzmann's constant which relates the microscopic energy scale with temperature. Using the known value of the electron charge ($e = 1.602 \times 10^{-19}$ C) Boltzmann's constant can be extracted from your measurements. You will also verify the Boltzmann distribution law $I \propto \exp(-eV/k_BT)$ at several temperatures T. Here I denotes current and V is voltage.

This experiment will require that you measure currents spanning five orders of magnitude, from tens of nano-amps to milli-amps. These measurements will be made using a simple 741 op-amp circuit, a device with which you became familiar in PHYS 231.

After completing a successful set of measurements at room temperature, you will use a water bath, ice, and a hotplate to repeat the experiment at at least two other temperatures.

2 Pre-lab Questions

Before attempting these pre-lab questions first read the entire lab manual, you may find some useful information that will be helpful when answering some of these questions.

1. Rearrange Eq. 2 given in the text below and show how Boltzmann's constant can be obtained from the slope of a straight line. Justify any approximations you make.

2. As mentioned above, you will use an op-amp circuit to measure current. Show that for the circuit pictured in Fig. 1 the current is given by $i_{in} = -V_{out}/R_f$.

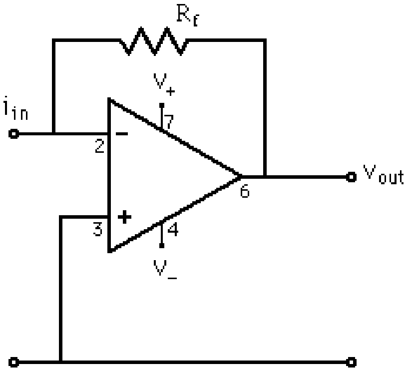

Figure 1: Op-amp circuit: current to voltage converter.

3 Semiconductor devices

The information presented in this section is based on a useful article entitled *Measurement of Boltzmann's constant* by D.E. Evans [1] which you are encouraged to read. You can find a copy of this article on the course website (`https://people.ok.ubc.ca/jbobowsk/phys232/manual/Evans.pdf`).

Diodes are devices made by "joining" a p-type semiconductor with an n-type semiconductor with an abrupt junction. An n-type semiconductor is made by doping a material (*eg.* silicon) with *donor* atoms that have one extra valence electron. Thus for each donor atom there will be one loosely bound electron introduced. Conversely, p-type semiconductors are doped with *acceptor* atoms that have one too few valence electrons and thus each acceptor atom introduces one *hole* that can propagate freely through the crystal. Abrupt junctions can be made by switching from donor atoms to acceptor atoms during the semiconductor crystal growth.

With no externally applied voltage across the diode, the net current flowing through the diode is zero. Near the diode junction a *depletion region* with a *contact potential* V_0 is created because electrons (holes) from the n-type (p-type) region diffuse across the junction and combine with holes (electrons) in the p-type (n-type) region (see Fig. 2). Once the depletion region is created, electrons in the n-type region can diffuse into the p-type region only if they have enough energy to overcome the contact potential energy eV_0. The number of electrons with sufficient energy is determined by the *Boltzmann distribution*, which says that at temperature T, the probability of an electron having energy eV_0 is given by: $\exp(-eV_0/k_BT)$. Thus, a *forward* electron current I from the n- to the p-type material is created and is given by:

$$I = A\exp(-eV_0/k_BT), \tag{1}$$

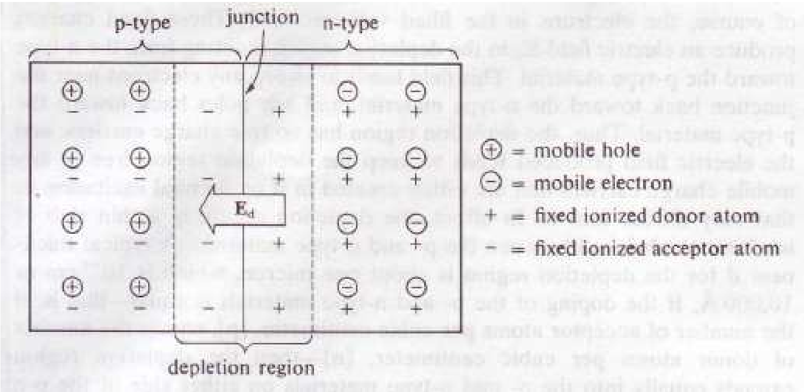

Figure 2: Depletion region of a diode. Figure taken from reference [2].

where A is a constant of proportionality. There is also a *reverse* electron current from the p- to n-type region. Electrons are created when bonds in the p-type region are broken due to thermal energy. If these created electrons make it into the depletion region they are swept into the n-type material causing current to flow. In equilibrium the two currents are equal and opposite to each other [1].

A diode is said to be forward biased if a voltage V is applied across the diode such that the p-type region is made positive with respect to the n-type region. In this case the electric field produced by the battery $\mathbf{E}_\mathrm{B}$ opposes the natural electric field $\mathbf{E}_\mathrm{d}$ due to the depletion region. See Fig. 3. The applied voltage alters the height of the potential barrier across the depletion region so that the forward electron current is proportional to $\exp[-e(V_0 - V)/k_BT]$. The reverse electron current is

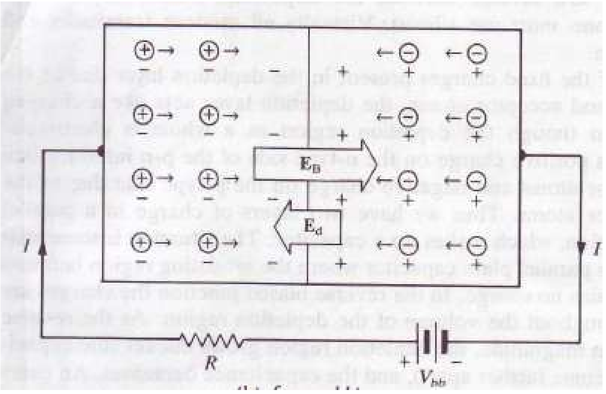

Figure 3: Forward biased diode. Figure taken from reference [2].

unchanged so that the net forward current is given by:

$$\begin{aligned} I &= A\mathrm{e}^{-e(V_0-V)/k_BT} - A\mathrm{e}^{-eV_0/k_BT} \\ &= A\mathrm{e}^{-eV_0/k_BT}\left(\mathrm{e}^{eV/k_BT}-1\right) \\ &= I_0\left(\mathrm{e}^{eV/k_BT}-1\right). \end{aligned} \tag{2}$$

Unfortunately, there are additional contributions to the diode current that we have neglected. When these additional currents are accounted for, the above relationship is modified such that:

$$I = I_0\left(\mathrm{e}^{eV/mk_BT}-1\right), \tag{3}$$

where m is a parameter that varies from diode to diode and is typically between 1 and 2.5. Because of these additional currents, diodes cannot be easily used to determine k_B [1].

Fortunately, Eq. 2 can be applied to a transistor circuit. An npn transistor is a p-type region sandwiched in between two n-type regions. The regions are termed *emitter*, *base*, and *collector*. The base is the central p-type region and the emitter and collector are the n-doped regions on either end. If we forward bias the base-emitter junction, then a forward current will flow. If the collector is maintained at the same potential as the base, then it is found that the collector current is described very accurately by Eq. 2 and the extra diode currents that cause the modification of Eq. 3 are drained through the base and do not interfere with the collector current.

4 Experimental Setup

The transistor used in this experiment is a TIP31A npn power transistor. Because the emitter is n-type and the base is p-type the base-emitter voltage $V_{\mathrm{BE}} = V_{\mathrm{B}} - V_{\mathrm{E}}$ must be positive to forward bias the base-emitter junction. Because the base is held at 0, V_{E} must be negative. The circuit used in this experiment is shown in Fig. 4. You will need to build the op-amp current detection circuit and you will also need to supply a variable negative voltage to the emitter of the transistor. One possible method using a variable resistor is shown in the figure. Typical emitter voltages that you will use are ≈ -0.5 V.

5 Procedure

You should first obtain and analyze a set of room temperature data to familiarize yourself with the experiment and analysis before proceeding to get data at other temperatures. Use a standard alcohol thermometer to measure the temperature of the room. You should build your circuits on a

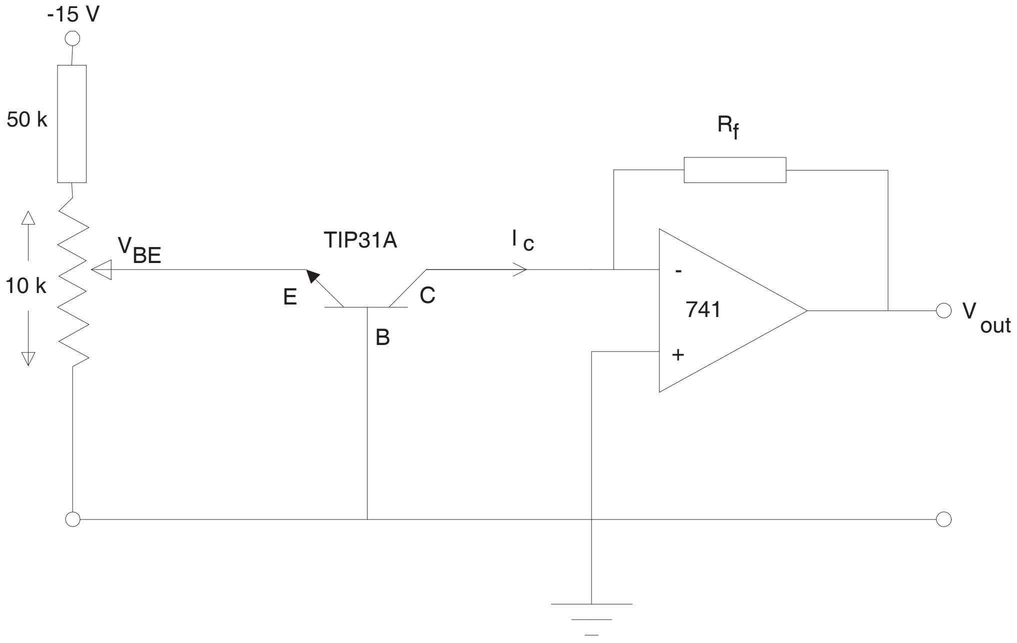

Figure 4: Circuit used to determine Boltzmann's constant.

breadboard board so that you can save for the next session

1. Begin by constructing the circuit shown in Fig. 4. Ensure that all power supplies are turned off when making electrical connections to the op-amp circuit. Don't forget that the op-amp needs ±15 V supplies!

2. Before using your circuit to collect data, you will need to measure the offset voltage at the output terminal of the op-amp. Start with a R_f=1 MΩ feedback resistor (which will allow you to measure currents from fractions of a microamp up to $\approx 15\ \mu$A). After carefully ensuring that the circuit is connected correctly, set $V_\mathrm{E} = 0$ V, turn on the ±15 V power to the op-amp. Now measure V_out. This value is the offset voltage. When making measurements of V_out with nonzero V_E, you will need to subtract the offset voltage from your measurement. For example, if you have an offset voltage of 0.209 V and then turn on V_E and measure 2.090 V at the output of the op-amp, the true $V_\mathrm{out} = (2.090 - 0.209)$ V = 1.881 V (make sure to account for propagation of errors each time you do this subtraction).

3. Now measure the collector current as a function of V_E. You must measure V_E very accurately so use the best digital voltmeter you can find. Start with small values of V_E and increase the voltage very slowly.

4. Once you have saturated the op-amp output (15 V) turn off all the power supplies and switch to a 100 kΩ feedback resistor. You will have to remeasure the op-amp offset voltage → repeat

steps (2) and (3) above.

5. Collect data sets for R_f=1 MΩ, 100 kΩ, 10 kΩ, and 1 kΩ. Using this set of feedback resistors will allow you to measure currents from fractions of a microamp to 10 mA. Four to five data points for each feedback resistor will be sufficient.

6. Analyze your data and extract a value for k_B.

Once you are satisfied with your room temperature results you may proceed with measurements at other temperatures set by the water bath.

The final value of Boltzmann's constant you report should be a weighted average (with an associated weighted error) of all of your measurements at different temperatures. Your final analysis should include a plot of $\ln I_\mathrm{C}$ versus V_BE with linear fits. You should plot all of your data sets on a single graph. That is, for each temperature combine all your I_C and V_BE from all feedback resistors and do one linear fit. Plot all data from all temperatures on a single plot and show the best-fit line determined from a weighted linear fit for each temperature.

References

[1] D. E. Evans. Measurement of Boltzmann's constant. *Phys. Educ.*, 21:296, 1986.

[2] R. E. Simpson. *Introductory Electronics for Scientists and Engineers 2nd Ed.* Prentice Hall, Inc., New Jersey, 1987.